

Sub-grid-scale convection modeling in basin-scale ocean circulation modeling

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Applying primitive equations to each density under the circumstances where density in each numerical cell is heterogeneous and where the number of fluid parcels with each density in each numerical cell is large enough for continuum approximation, I modeled sub-grid scale convection in basin-scale ocean circulation modeling. In each numerical cell, the relationship among salinity, potential temperature, and specific volume or reciprocal of density and that between velocity and specific volume are linearly approximated. Here, equations are formulated for proportional ratios between perturbations of velocity and specific volume in each cell to evolve 4-dimensionally. To save computational tasks as much as possible, it is recommended to adopt models which does not deal with probability distribution in each cell as a function of specific volume but evolves variance of specific volume 4-dimensionally. This model, based on dependence of inertia on density, calculates not only vertical convection but also variance of horizontal velocity by density in each cell. The vertical momentum equation explicitly takes buoyancy into consideration.

Keywords : convection, ocean circulation model, basin scale, density, heterogeneous

海盆スケールの海洋循環モデルにおける サブグリッドスケールの対流のモデル化

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各計算格子内で様々な密度の流体粒子が混在して、かつ各密度の流体粒子の数が連続体近似をするのに十分なほど多い状況で、密度別にprimitive方程式を適用することにより海盆スケールの海洋循環モデルにおけるサブグリッドスケールの対流をモデル化した。各計算格子内では密度の逆数である比体積と塩分と温位の間の関係及び比体積と流速の間の関係を線形近似した。ここで比体積と流速それぞれの計算格子内での平均からの変動の間の比例係数が4次元的に時間発展する方程式を定式化した。計算量を極力節約するため、各計算格子内で流体粒子の確率密度分布を比体積の関数として与える代わりに、比体積の分散が4次元的に時間発展するようなモデルを採用する。本モデルは慣性の密度依存に基づくので、鉛直対流のみならず、各計算格子内での水平流速の密度によるばらつきも計算する。鉛直方向の方程式の定式化では浮力も明示的に考慮した。

キーワード：対流，海洋循環モデル，海盆スケール，密度，不均一

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1. Introduction

In the Arctic and Southern Oceans, sea ice is created in winter to generate dense brine water. The brine sinks to a certain depth where its buoyancy is 0, mixed with ambient water rather slowly. Such phenomena cannot be resolved at all in basin-scale ocean circulation models, because of fluid parcels of brine is much smaller than numerical cells. Thus, some parameterization is necessary. This paper discusses the circumstances where density in each numerical cell is heterogeneous and where the number of fluid parcels with each density in each numerical cell is large enough for continuum approximation. Momentum equations of fluid parcels with specific volume α are as follows;

$$\frac{\partial w}{\partial t} + (\vec{u} \cdot \nabla) w + \frac{1}{2} \frac{\partial}{\partial z} (w^2) = -\alpha \frac{\partial p}{\partial z} - g + \nabla \cdot \mathbf{v}_h \nabla w + \frac{\partial}{\partial z} v_v \frac{\partial w}{\partial z} \quad (1.1)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + w \frac{\partial \vec{u}}{\partial z} + f \mathbf{k} \times \vec{u} = -\alpha \nabla p + \nabla \cdot \mathbf{v}_h \nabla \vec{u} + \frac{\partial}{\partial z} v_v \frac{\partial \vec{u}}{\partial z} \quad (1.2)$$

Here, arrows denote 2-dimensional vectors; $\mathbf{k}$: unit z vector, $\mathbf{u}$: horizontal velocity, w : vertical velocity; p : pressure, α : specific volume of water (reciprocal of density), f : Coriolis parameter, g : gravity acceleration, t : time, z : vertical coordinate, $\mathbf{v}$: coefficient of eddy kinematic viscosity, where the suffixes denote horizontal and vertical, respectively. In this paper, the upward direction is positive in the z axis, where $z = \zeta$ at the surface and $z = -H$ at the bottom. Note that the bottom topography H is positive.

The continuity equation is as follows;

$$\nabla \cdot \vec{u} + \frac{\partial w}{\partial z} = 0 \quad (1.3)$$

Frontier Research System for Global Change (FRSGC) aims at horizontal resolution of 10 km. In linear systems, output is proportional to input. Thus, output is averaged when input is also averaged. Thus, it is not linear but non-linear terms that create essential change with higher resolution. Thus, this paper does not omit advection terms. The input of ocean circulation models is boundary conditions.

The following sections successively discuss buoyancy term, linear approximation between velocity and spe-

cific volume in each cell, ensemble mean momentum equations, and variance of specific volume in each cell.

2. Notation definitions and hydrostatic balance

To derive a buoyancy term from Eq. (1.1), the hydrostatic pressure p_H is defined by the following equations:

$$\frac{\partial p_H}{\partial z} \equiv -\frac{g}{\alpha} \quad (2)$$

In this paper, upper bars as over α in the above equation refer to ensemble mean in a numerical cell, while primes to perturbations from the mean.

$$\alpha \equiv \bar{\alpha} + \alpha' \quad (3)$$

$$w \equiv \bar{w} + w' \quad (4.1)$$

$$\vec{u} \equiv \bar{\vec{u}} + \vec{u}' \quad (4.2)$$

Here, note that velocities in the left hand side and perturbations in the right had side are regarded as functions of not only x, y, z , and t but also of α , while ensemble means in each cell are only of x, y, z , and t . See Figure 1.

The hydrostatic pressure can be straightforwardly determined by integration of Eq. (2) from depth z to the surface. Now, the non-hydrostatic pressure is defined by the following equation:

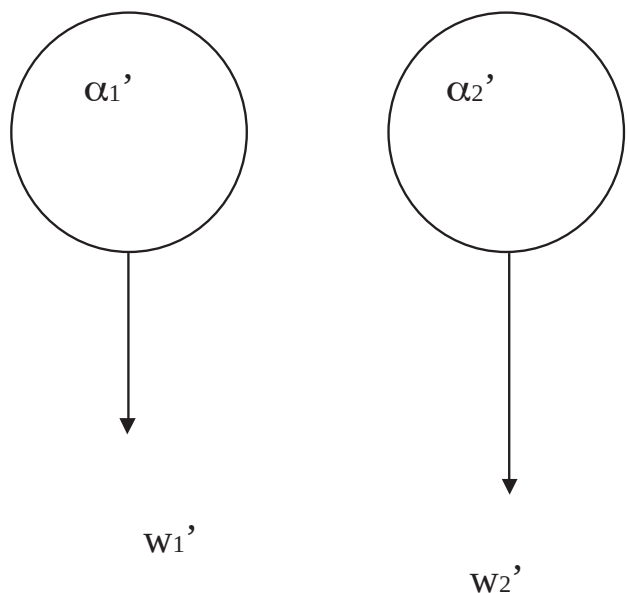


Fig. 1 Specific volume and velocity of fluid parcel. Velocity depends on specific volume in each cell.

$$p_N \equiv p - p_H \quad \dots\dots\dots (5)$$

Substitution of Eqs. (3) and (5) into (1.1) and (1.2) yields:

$$\begin{aligned} \frac{\partial w}{\partial t} + (\vec{u} \cdot \nabla) w + \frac{1}{2} \frac{\partial}{\partial z} (w^2) \\ = -\alpha \frac{\partial p_N}{\partial z} + \frac{\alpha'}{\alpha} g + \nabla \cdot v_h \nabla w + \frac{\partial}{\partial z} v_v \frac{\partial w}{\partial z} \quad \dots\dots\dots (6.1) \end{aligned}$$

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + w \frac{\partial \vec{u}}{\partial z} + f \vec{k} \times \vec{u} \\ = -\alpha \nabla (p_N + p_H) + \nabla \cdot v_h \nabla \vec{u} + \frac{\partial}{\partial z} v_v \frac{\partial \vec{u}}{\partial z} \quad \dots\dots\dots (6.2) \end{aligned}$$

Here, unknown are p_N and Euler's differentials of u and w . Principally, Eqs. (6) and (1.3) can be solved together to forward velocities in time. Note that the second term in the right hand side of Eq. (6.1) is buoyancy.

The following section relates velocity with specific volume linearly in each cell.

3. Linear approximation between velocity and specific volume

In each numerical cell, the following linear approximation is adopted to regard the partial differentials in the following equations as functions not of α but only of x , y , z , and t :

$$w' \approx \alpha' \frac{\partial w}{\partial \alpha} \Big|_{\alpha=\bar{\alpha}} \quad \dots\dots\dots (7.1)$$

$$\vec{u}' \approx \alpha' \frac{\partial \vec{u}}{\partial \alpha} \Big|_{\alpha=\bar{\alpha}} \quad \dots\dots\dots (7.2)$$

Henceforth, suffixes of the above coefficients, which are functions of x , y , z , and t , will be omitted in this paper. The advantage of this linear approximation will be shown in the later sections.

Partial differentiation of Eqs. (6) by α yields the following equations:

$$\begin{aligned} \frac{\partial^2 w}{\partial t \partial \alpha} + \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) w + (\vec{u} \cdot \nabla) \frac{\partial w}{\partial \alpha} + \frac{\partial w}{\partial \alpha} \frac{\partial w}{\partial z} + w \frac{\partial^2 w}{\partial z \partial \alpha} \\ = -\frac{\partial p_N}{\partial z} + \frac{g}{\alpha} + \nabla \cdot v_h \nabla \frac{\partial w}{\partial \alpha} + \frac{\partial}{\partial z} v_v \frac{\partial^2 w}{\partial z \partial \alpha} \quad \dots\dots\dots (8.1) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \vec{u}}{\partial t \partial \alpha} + \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) \vec{u} + (\vec{u} \cdot \nabla) \frac{\partial \vec{u}}{\partial \alpha} + \frac{\partial w}{\partial \alpha} \frac{\partial \vec{u}}{\partial z} + w \frac{\partial^2 \vec{u}}{\partial z \partial \alpha} + f \vec{k} \times \frac{\partial \vec{u}}{\partial \alpha} \\ = -\nabla (p_N + p_H) + \nabla \cdot v_h \nabla \frac{\partial \vec{u}}{\partial \alpha} + \frac{\partial}{\partial z} v_v \frac{\partial^2 \vec{u}}{\partial z \partial \alpha} \quad \dots\dots\dots (8.2) \end{aligned}$$

Partial differentiation of Eq. (1.3) by α yields the following equation:

$$\nabla \cdot \frac{\partial \vec{u}}{\partial \alpha} + \frac{\partial^2 w}{\partial z \partial \alpha} = 0 \quad \dots\dots\dots (8.3)$$

Substitution of Eqs. (4) and (7) and $\alpha' = 0$ into (8.1) and (8.2) yields the following equations:

$$\begin{aligned} \frac{\partial^2 w}{\partial t \partial \alpha} + \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) \bar{w} + (\bar{\vec{u}} \cdot \nabla) \frac{\partial w}{\partial \alpha} + \frac{\partial w}{\partial \alpha} \frac{\partial \bar{w}}{\partial z} + \bar{w} \frac{\partial^2 w}{\partial z \partial \alpha} \\ = -\frac{\partial p_N}{\partial z} + \frac{g}{\alpha} + \nabla \cdot v_h \nabla \frac{\partial w}{\partial \alpha} + \frac{\partial}{\partial z} v_v \frac{\partial^2 w}{\partial z \partial \alpha} \quad \dots\dots (9.1) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \vec{u}}{\partial t \partial \alpha} + \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) \bar{\vec{u}} + (\bar{\vec{u}} \cdot \nabla) \frac{\partial \vec{u}}{\partial \alpha} + \frac{\partial w}{\partial \alpha} \frac{\partial \bar{\vec{u}}}{\partial z} + \bar{w} \frac{\partial^2 \vec{u}}{\partial z \partial \alpha} + f \vec{k} \times \frac{\partial \vec{u}}{\partial \alpha} \\ = -\nabla (p_N + p_H) + \nabla \cdot v_h \nabla \frac{\partial \vec{u}}{\partial \alpha} + \frac{\partial}{\partial z} v_v \frac{\partial^2 \vec{u}}{\partial z \partial \alpha} \quad \dots\dots\dots (9.2) \end{aligned}$$

In short, given ensemble mean velocities and proper initial and boundary conditions, the coefficients, which are partial differentials of velocities by α , can be forwarded in time. As boundary conditions at the sea surface and the bottom, for example, the following equations may be adopted.

$$\begin{aligned} \frac{\partial w}{\partial \alpha} &= 0 \\ \text{and} \\ \frac{\partial \vec{u}}{\partial \alpha} &= \vec{0} \quad \dots\dots\dots (10) \end{aligned}$$

The following section discusses how to forward ensemble mean velocities in time.

4. Ensemble mean momentum equations

Substitution of Eqs. (7) into ensemble mean of Eqs. (6) yields the following equations:

$$\begin{aligned} \frac{\partial \bar{w}}{\partial t} + (\bar{\vec{u}} \cdot \nabla) \bar{w} + \overline{(\alpha')^2} \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) \frac{\partial w}{\partial \alpha} + \frac{1}{2} \frac{\partial}{\partial z} (\bar{w}^2) + \frac{\overline{(\alpha')^2}}{2} \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial \alpha} \right)^2 \\ = -\bar{\alpha} \frac{\partial p_N}{\partial z} + \nabla \cdot v_h \nabla \bar{w} + \frac{\partial}{\partial z} v_v \frac{\partial \bar{w}}{\partial z} \quad \dots\dots\dots (11.1) \end{aligned}$$

$$\begin{aligned} \frac{\partial \bar{\vec{u}}}{\partial t} + (\bar{\vec{u}} \cdot \nabla) \bar{\vec{u}} + \overline{(\alpha')^2} \left(\frac{\partial \vec{u}}{\partial \alpha} \cdot \nabla \right) \frac{\partial \vec{u}}{\partial \alpha} + \bar{w} \frac{\partial \bar{\vec{u}}}{\partial z} + \overline{(\alpha')^2} \frac{\partial w}{\partial \alpha} \frac{\partial^2 \vec{u}}{\partial z \partial \alpha} + f \vec{k} \times \bar{\vec{u}} \\ = -\bar{\alpha} \nabla (p_N + p_H) + \nabla \cdot v_h \nabla \bar{\vec{u}} + \frac{\partial}{\partial z} v_v \frac{\partial \bar{\vec{u}}}{\partial z} \quad \dots\dots\dots (11.2) \end{aligned}$$

$$\nabla \cdot \bar{\vec{u}} + \frac{\partial \bar{w}}{\partial z} = 0 \quad \dots\dots\dots (11.3)$$

Thus, once variance of specific volume α is given, ensemble mean velocities and α -partial differentials of velocities can be forwarded in time through Eqs. (8.3), (9), and (11).

5. Variance of specific volume

Specific volume varies with a certain probability distribution in each cell. However, it requires too much computational power to calculate the probability distribution directly. Instead, to save the computational tasks as much as possible, only variance of specific volume is calculated.

Since specific volume α is a function of salinity S , potential temperature Θ , and pressure, their relationship in a numerical cell is approximated in the following way;

$$\alpha' \approx S' \frac{\partial \alpha}{\partial S} \Big|_{\alpha=\bar{\alpha}} + \Theta' \frac{\partial \alpha}{\partial \Theta} \Big|_{\alpha=\bar{\alpha}} \quad \dots\dots\dots (12)$$

Here, influence of pressure on specific volume is neglected, because much less important in convection. Then,

$$\overline{(\alpha')^2} \approx \overline{(S')^2} \left(\frac{\partial \alpha}{\partial S} \Big|_{\alpha=\bar{\alpha}} \right)^2 + 2 \overline{S' \Theta'} \frac{\partial \alpha}{\partial S} \Big|_{\alpha=\bar{\alpha}} \frac{\partial \alpha}{\partial \Theta} \Big|_{\alpha=\bar{\alpha}} + \overline{(\Theta')^2} \left(\frac{\partial \alpha}{\partial \Theta} \Big|_{\alpha=\bar{\alpha}} \right)^2 \quad \dots\dots\dots (13)$$

Thus, once variances of salinity and potential temperature are given, variance of specific volume is also determined. To obtain variances of S and Θ , for example, the Level 3 turbulence-closure model of Mellor and Yamada (1974) can be adopted, because the variances evolve 4-dimensionally through mixing and advection. At the sea bottom, these variances can be set, for example, to 0 as boundary conditions. The turbulence closure model does not account for fluctuating components of density or reciprocal of specific volume but for variances of salinity and temperature, except for thermal expansion (Mellor and Yamada, 1982). In short, density is homogeneous in a numerical cell. On the other hand, this paper assumes that density is heterogeneous in a cell, accompanied by sub-grid scale convection. Nonetheless, the turbulence closure model is recommended to be adopted for evolution of variances of salinity and temperature although influence of density heterogeneity on turbulence remains to be studied. For example, terms of friction among particles with different specific volume may be desired to be added to Eqs. (6). However, turbulence may be superior to such friction.

6. Orthogonal σ coordinate system

Basin-scale ocean circulation models, first, integrate momentum equations vertically from the surface to the bottom, to save computational time with a longer time step, and then solve original momentum equations with a shorter time step to determine velocity in each layer. To avoid basal flow terms derived from order exchange of integral and differential operators in momentum equations integrated vertically from the surface to the bottom, Kakuta (in this volume) proposed formulation of momentum equations directly in the orthogonal σ -coordinate system, instead of transformation from the Cartesian coordinate system to σ . This section demonstrates application of this convection modeling to the orthogonal σ coordinate system.

The orthogonal σ -coordinate-system equations corresponding to Eqs. (1.3) and (6) can be written as follows;

$$\begin{aligned} & \frac{\partial w_\sigma}{\partial t} + (\vec{v} \cdot \nabla) w_\sigma + \frac{1}{2(\zeta + H)} \frac{\partial}{\partial \sigma} (w_\sigma^2) + f \sigma \vec{k} \cdot (\vec{v} \times \nabla H) \\ &= - \frac{\alpha}{\zeta + H} \frac{\partial p_N}{\partial \sigma} + \frac{\alpha'}{\bar{\alpha}} g + \nabla \cdot \mathbf{v}_h \nabla w_\sigma + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial w_\sigma}{\partial \sigma} \quad \dots\dots\dots (14.1) \end{aligned}$$

$$\begin{aligned} & \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} + \frac{1}{\zeta + H} w_\sigma \frac{\partial \vec{v}}{\partial \sigma} + f \vec{k} \times \vec{v} \\ &= - \alpha \nabla (p_N + p_H) + g \sigma \nabla H + \nabla \cdot \mathbf{v}_h \nabla \vec{v} + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial \vec{v}}{\partial \sigma} \quad \dots\dots\dots (14.2) \end{aligned}$$

$$\nabla \cdot \vec{v} + \frac{1}{\zeta + H} \frac{\partial w_\sigma}{\partial \sigma} = 0 \quad \dots\dots\dots (14.3)$$

Here, the ∇ operators in Eq. (14.3) and in the pressure term in Eq. (14.1) and the first ∇ operators in the viscosity terms are not straightforward as seen in Appendix 1. Here, w_σ is σ component of velocity, not scaled by $(\zeta + H)$. Note that bottom gradients appear in the gravity term of the momentum equations in constant- σ surfaces and in the Coriolis term in constant- X and Y lines normal to constant- σ surfaces. In the hydrostatic balance and in the case of no density perturbations, the pressure and gravity terms in the right hand side of Eq. (14.2) reduce to horizontal pressure gradients multiplied by specific volume as seen in Appendix 2.

Thus, the orthogonal σ -coordinate-system equations corresponding to Eqs. (8.3) and (9) can be written as follows;

$$\begin{aligned} & \frac{\partial^2 w_\sigma}{\partial t \partial \alpha} + \left(\frac{\partial \vec{v}}{\partial \alpha} \cdot \nabla \right) \vec{w}_\sigma + (\vec{v} \cdot \nabla) \frac{\partial w_\sigma}{\partial \alpha} \\ & + \frac{1}{\zeta + H} \left(\frac{\partial w_\sigma}{\partial \alpha} \frac{\partial \vec{w}_\sigma}{\partial \sigma} + \vec{w}_\sigma \frac{\partial^2 w_\sigma}{\partial \sigma \partial \alpha} \right) + f \sigma \vec{k} \cdot (\vec{v} \times \nabla H) \\ & = - \frac{1}{\zeta + H} \frac{\partial p_N}{\partial \sigma} + \frac{g}{\alpha} + \nabla \cdot v_h \nabla \frac{\partial w_\sigma}{\partial \alpha} + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial^2 w_\sigma}{\partial \sigma \partial \alpha} \\ & \dots \dots \dots (15.1) \end{aligned}$$

$$\begin{aligned} & \frac{\partial^2 \vec{v}}{\partial t \partial \alpha} + \left(\frac{\partial \vec{v}}{\partial \alpha} \cdot \nabla \right) \vec{v} + (\vec{v} \cdot \nabla) \frac{\partial \vec{v}}{\partial \alpha} \\ & + \frac{1}{\zeta + H} \left(\frac{\partial w_\sigma}{\partial \alpha} \frac{\partial \vec{v}}{\partial \sigma} + \vec{w}_\sigma \frac{\partial^2 \vec{v}}{\partial \sigma \partial \alpha} \right) + f \vec{k} \times \vec{v} \\ & = - \nabla (p_N + p_H) + \nabla \cdot v_h \nabla \frac{\partial \vec{v}}{\partial \alpha} + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial^2 \vec{v}}{\partial \sigma \partial \alpha} \\ & \dots \dots \dots (15.2) \end{aligned}$$

$$\nabla \cdot \frac{\partial \vec{v}}{\partial \alpha} + \frac{1}{\zeta + H} \frac{\partial^2 w_\sigma}{\partial \sigma \partial \alpha} = 0 \quad \dots \dots \dots (15.3)$$

Note that the gravity term disappears in Eq. (15.2). On the other hand, the orthogonal σ -coordinate-system equations corresponding to Eqs. (11) can be written as follows;

$$\begin{aligned} & \frac{\partial \vec{w}_\sigma}{\partial t} + (\vec{v} \cdot \nabla) \vec{w}_\sigma + (\alpha')^2 \left(\frac{\partial \vec{v}}{\partial \alpha} \cdot \nabla \right) \frac{\partial w_\sigma}{\partial \alpha} \\ & + \frac{1}{2(\zeta + H)} \left\{ \frac{\partial}{\partial \sigma} (\vec{w}_\sigma^2) + (\alpha')^2 \frac{\partial}{\partial \sigma} \left(\frac{\partial w_\sigma}{\partial \alpha} \right)^2 \right\} + f \sigma \vec{k} \cdot (\vec{v} \times \nabla H) \\ & = - \frac{\bar{\alpha}}{\zeta + H} \frac{\partial p_N}{\partial \sigma} + \nabla \cdot v_h \nabla \vec{w}_\sigma + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial \vec{w}_\sigma}{\partial \sigma} \\ & \dots \dots \dots (16.1) \end{aligned}$$

$$\begin{aligned} & \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} + (\alpha')^2 \left(\frac{\partial \vec{v}}{\partial \alpha} \cdot \nabla \right) \frac{\partial \vec{v}}{\partial \alpha} \\ & + \frac{1}{\zeta + H} \left\{ \vec{w}_\sigma \frac{\partial \vec{v}}{\partial \sigma} + (\alpha')^2 \frac{\partial w_\sigma}{\partial \alpha} \frac{\partial^2 \vec{v}}{\partial \sigma \partial \alpha} \right\} + f \vec{k} \times \vec{v} \\ & = - \bar{\alpha} \nabla (p_N + p_H) + g \sigma \nabla H + \nabla \cdot v_h \nabla \vec{v} + \frac{1}{(\zeta + H)^2} \frac{\partial}{\partial \sigma} v_v \frac{\partial \vec{v}}{\partial \sigma} \\ & \dots \dots \dots (16.2) \end{aligned}$$

$$\nabla \cdot \vec{v} + \frac{1}{\zeta + H} \frac{\partial \vec{w}_\sigma}{\partial \sigma} = 0 \quad \dots \dots \dots (16.3)$$

Note that the buoyancy term disappears in Eq. (16.1). The above equations can be forwarded in time with initial and boundary conditions.

7. Conclusions

Applying primitive equations to each density under the circumstances where density in each numerical cell is heterogeneous and where the number of fluid parcels with each density in each numerical cell is large enough for continuum approximation, I modeled sub-grid scale convection in basin-scale ocean circulation modeling. In each numerical cell, the relationship among salinity, potential temperature, and specific volume or reciprocal

of density and that between velocity and specific volume are linearly approximated. Here, equations are formulated for proportional ratios between perturbations of velocity and specific volume in each cell to evolve 4-dimensionally. To save computational tasks as much as possible, it is recommended to adopt models which does not deal with probability distribution in each cell as a function of specific volume but evolves variance of specific volume 4 dimensionally. Since density is a function of salinity and potential temperature, variances of salinity and potential temperature gives specific volume variance. Variances of salinity and potential temperature is determined, for example, by Level 3 of the turbulence closure model of Mellor and Yamada (1974).

This model proposed in this paper, based on dependence of inertia on density, calculates not only vertical convection but also variance of horizontal velocity by density in each cell. The vertical momentum equation explicitly takes buoyancy into consideration. To avoid basal flow terms derived from order exchange of integral and differential operators in momentum equations integrated vertically from the surface to the bottom, equations are reformulated directly in the orthogonal σ -coordinate system.

Appendix 1

In the orthogonal σ coordinate system, the X direction gradient of pressure force exerting on a volume element can be written as follows;

$$\begin{aligned} & \frac{(p + \Delta p) \left(H + \frac{\partial H}{\partial X} \Delta X \right) \Delta \sigma \Delta Y - p H \Delta \sigma \Delta Y}{\left(H + \frac{1}{2} \frac{\partial H}{\partial X} \Delta X \right) \Delta \sigma \Delta Y \Delta X} \\ & = \frac{1}{H \Delta X} \left(p \frac{\partial H}{\partial X} \Delta X + H \Delta p + \frac{\partial H}{\partial X} \Delta X \Delta p \right) \\ & \rightarrow \frac{1}{H} \left(\frac{\partial H}{\partial X} p + H \frac{\partial p}{\partial X} \right) = \frac{1}{H} \frac{\partial}{\partial X} (H p) \quad \dots \dots \dots (A3.1) \end{aligned}$$

when $\Delta X \rightarrow 0$. Here, pressure is p at a coordinate X and $p + \Delta p$ at $X + \Delta X$. Here, X can be regarded as identical to x , when squares of bottom gradients are neglected (Kakuta, in this volume). For simplicity of notation, surface height ζ is omitted. See Figure 2 for visual comprehension. Similarly, the Y direction gradient of pressure force can be considered. Thus, the definition of the ∇ operator in the pressure term in the momentum equations in orthogonal σ coordinate system is as follows;

$$\nabla = \left(\frac{1}{H} \frac{\partial H}{\partial X} + \frac{\partial}{\partial X}, \frac{1}{H} \frac{\partial H}{\partial Y} + \frac{\partial}{\partial Y} \right) \quad \dots\dots\dots (A3.2)$$

If surface height ζ is not omitted,

$$\nabla = \left(\frac{1}{\zeta+H} \frac{\partial}{\partial X} (\zeta+H) + \frac{\partial}{\partial X}, \frac{1}{\zeta+H} \frac{\partial}{\partial Y} (\zeta+H) + \frac{\partial}{\partial Y} \right) \quad \dots\dots\dots (A3.3)$$

The first ∇ operator in viscosity terms is also defined as above, similarly in consideration of gradients of stress force exerting on a volume element. The ∇ operator in the continuity equation in orthogonal σ coordinate system is also defined by (A3.3). On the other hand, the definition of the other ∇ operators: ∇ in advection, gravity, and Coriolis terms and the second ∇ in viscosity terms is straightforward, namely,

$$\nabla = \left(\frac{\partial}{\partial X}, \frac{\partial}{\partial Y} \right) \quad \dots\dots\dots (A3.4)$$

Appendix 2

Substitution of Eqs. (2) and (3) into the pressure and gravity terms in the right hand side of Eq. (14.2) yields the following terms;

$$\begin{aligned} & -\alpha \nabla (p_H + p_N) - g \sigma \nabla H \\ & = -\bar{\alpha} \nabla \left(p_H + \frac{\partial p_H}{\partial \sigma} \frac{\sigma}{\zeta+H} \nabla H \right) - \alpha' \nabla p_H - \alpha \nabla p_N \dots (A.4) \end{aligned}$$

In the hydrostatic balance and in the case of no density perturbations, the last 2 terms disappear. In the last parentheses, horizontal gradients of pressure appear. Note that this equation is written in the orthogonal σ -coordinate system.

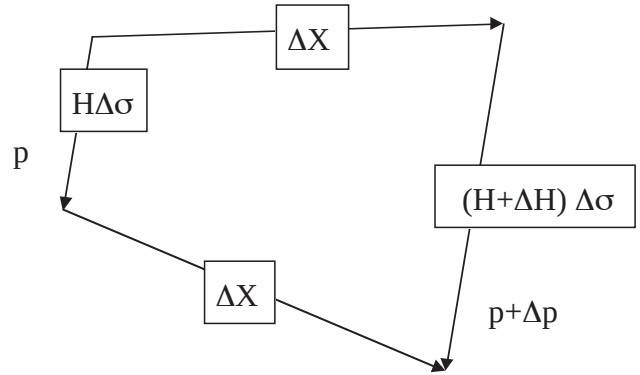


Fig. 2 Volume element in the orthogonal- σ coordinate system. Y direction is normal to the sheet of paper, and thus omitted. Pressure is also shown. Force by pressure is pressure multiplied by element area. The element areas are different between the left and right side of the volume element. In momentum equations, the difference in force between the both sides is divided by volume element.

Here,

$$\Delta H = \frac{\partial H}{\partial X} \Delta X$$

$$\Delta p = \frac{\partial p}{\partial X} \Delta X$$

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